fMRI unmixing via properly adjusted Dictionary Learning

Yannis Kopsinis[1], Harris Georgiou[1], Sergios Theodoridis[1]

[1]University of Athens, Dept. of Informatics and Telecommunications, Athens, Greece.

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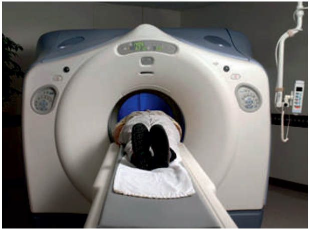
Outline

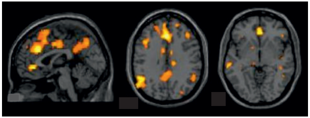
1. functional Magentic Resonance Imaging (fMRI)
2. fMRI data unmixing
3. Dictionary based fMRI data unmixing
4. Performance using synthetic data

functional Magnetic Resonance Imaging (fMRI)

Function

- A Non-invasive tool for detecting brain activity along time
- exploits the different magnetic properties of oxygen-saturated versus oxygen-desaturated hemoglobin
- It detects localized changes in the hemodynamic flow of oxygenated blood
- Indirect way to measure brain activity.
- Aim: To "understand" the brain.

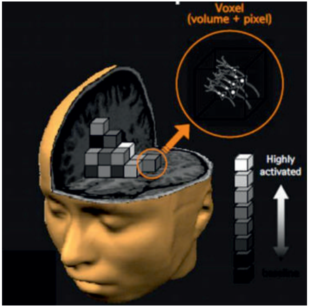


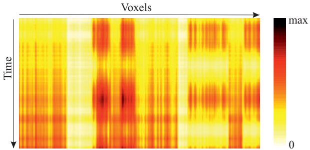


functional Magnetic Resonance Imaging (fMRI)

fMRI Data

- Typical Sampling characteristics: 3-D grid of 3-5 mm of elementary 'cubes' (*voxels*), typically 64x64x48 voxels per time instance, is acquired every 1-2 seconds.
- The full amount of data is collected in a data matrix $\boldsymbol{Y} \in \mathbb{R}^{t,n}$. Each column of $\boldsymbol{Y}$ represents the evolution in time of the values of a certain voxel.

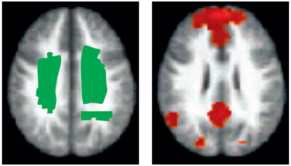




Functional Brain Networks (FBNs)

Characterization

- Comprise a number of *segregated* specialized small brain regions
- Each FBN forms a Spatial Map.
- Exhibit strong functional connectivity, which is expressed as strong coherence in the activation time-pattern Time Course.
- They are related to low-level brain functions.
- Examples of FBNs: the visual, sensorimotor, auditory, default-mode, dorsal attention



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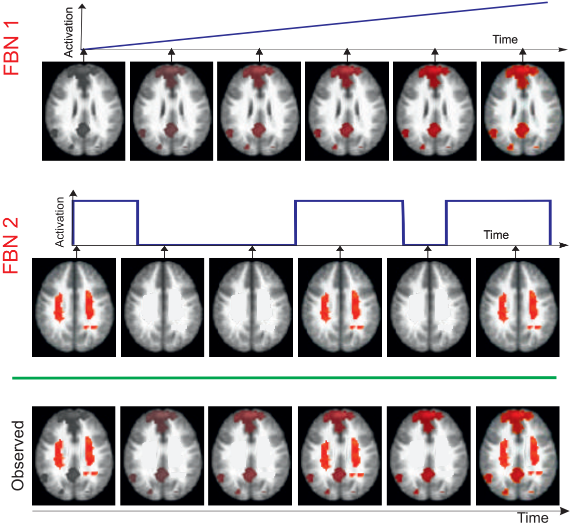


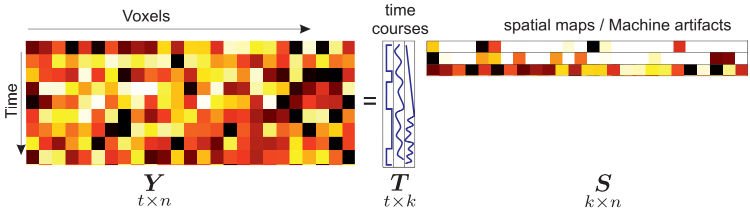
Functional Brain Networks (FBNs)

Mathematical modeling

$$\boldsymbol{Y} = \boldsymbol{T}_{\cdot,1}\boldsymbol{S}_{1,\cdot} + \boldsymbol{T}_{\cdot,2}\boldsymbol{S}_{2,\cdot} + \ldots + \boldsymbol{T}_{\cdot,k}\boldsymbol{S}_{k,\cdot} = \sum_{i=1}^{k}\boldsymbol{T}_{\cdot,i}\boldsymbol{S}_{i,\cdot} = \boldsymbol{T}\boldsymbol{S},$$

where $\boldsymbol{T}$, $\boldsymbol{S}$ are $(t \times k)$ and $(k \times n)$ matrices respectively whereas the columns $\boldsymbol{T}_{\cdot,i}$ representing time courses and the rows $\boldsymbol{S}_{i,\cdot}$ represents spatial maps.

- Spatial maps are Sparse
- Artifacts related to the acquisition process and machine-induced artifacts can be modeled in the same way, however they are Dense.



fMRI Unmixing

General Task

- Factor the data matrix as $\boldsymbol{Y} \approx \boldsymbol{A}\boldsymbol{B}$ aiming at getting $\boldsymbol{A} \approx \boldsymbol{T}$ and $\boldsymbol{B} \approx \boldsymbol{S}$
- There is an infinite number of such factorizations
- A priori information information about the characteristics of $\boldsymbol{B}$ and/or $\boldsymbol{A}$ need to get imposed

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fMRI unmixing approaches

- **Conventional fMRI unmixing via ICA:** Factor $\boldsymbol{Y} \approx \boldsymbol{A}\boldsymbol{B}$ under the constraint that the columns of $\boldsymbol{B}$ are realizations of a random variable having independent components, e.g. [Calhoun 2006]
- **Recent advances:** Factor $\boldsymbol{Y} \approx \boldsymbol{A}\boldsymbol{B}$ under the constraint that $\boldsymbol{B}$ exhibits certain structured sparsity characteristics, e.g. [Varoquaux 2011], [Lee 2011], [Abraham 2013], [Abolghasemi 2013].

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fMRI unmixing approaches

- **fMRI unmixing via Dictionary Learning:** Factor $\boldsymbol{Y} \approx \boldsymbol{A}\boldsymbol{B}$ under the constraint that the columns of $\boldsymbol{B}$ are **sparse**.

$$\min_{\boldsymbol{A},\boldsymbol{B}} \|\boldsymbol{Y} - \boldsymbol{A}\boldsymbol{B}\|_{\mathsf{F}}^2, \text{ s.t. } \|\boldsymbol{B}_{\cdot,j}\|_0 \leq K, \; j = 1 \cdots n, \quad (1)$$

- K-SVD: One of the best performing DL method.
- **Fast and Incoherent Dictionary Learning** (FIDL), [Abolghasemi 2013]:
  1. Incoherence is imposed in $\boldsymbol{A}$,
  2. relatively low complexity per iteration,
  3. improved performance to both synthetic and real fMRI data.

Dictionary Learning

K-SVD steps

Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are estimated via iteratively repeating two processing stages:

First Stage Keep $\boldsymbol{A}$ fixed and update $\boldsymbol{B}$, column by column, via sparse codings of $\boldsymbol{Y}_{\cdot,j}$, $\forall j$, i.e.,

$$\min_{\boldsymbol{B}_{\cdot,j}} \|\boldsymbol{Y}_{\cdot,j} - \boldsymbol{A}\boldsymbol{B}_{\cdot,j}\|_F^2 \text{ subject to } \|\boldsymbol{B}_{\cdot,j}\|_0 \leq K, \ \forall j$$

Approximate solution: OMP

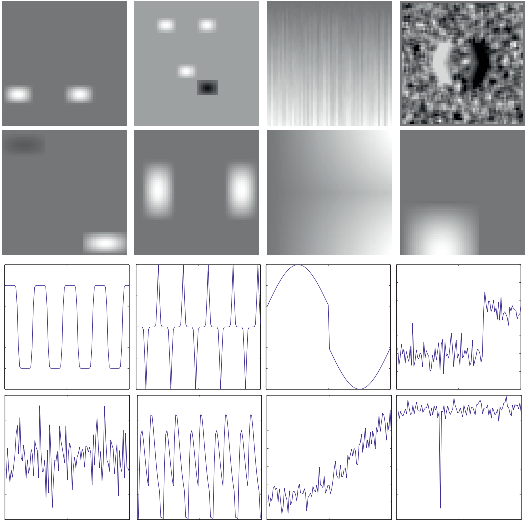
Second Stage Keep the support of $\boldsymbol{B}$ fixed and update sequentially the column-row pairs $\boldsymbol{A}_{\cdot,j}$ and $\boldsymbol{B}_{j,\cdot}$ to reduce $\|\boldsymbol{Y} - \boldsymbol{A}\boldsymbol{B}\|_F^2$

$$\min_{\boldsymbol{a},\boldsymbol{b}} \|\boldsymbol{E} + \boldsymbol{A}_{\cdot,j}\boldsymbol{B}_{j,\cdot} - \boldsymbol{a}\boldsymbol{b}^T\|_F^2 \text{ s.t. } \operatorname{supp}(\boldsymbol{B}_{j,\cdot})$$

Closed form solution: truncated SVD.

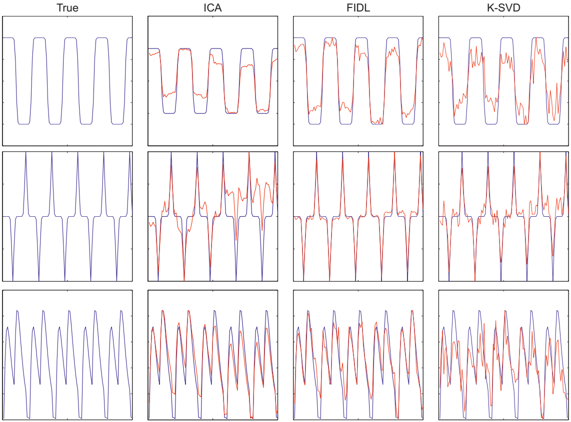
Synthetic fMRI data

[N. Correa, T. Adali, Yi-Ou Li, and V.D. Calhoun - 2005]



Some results

	Ca	Cm	Cam
fastICA	0.776	0.849	0.812
FIDL	0.836	0.792	0.814
K-SVD	0.798	0.697	0.747



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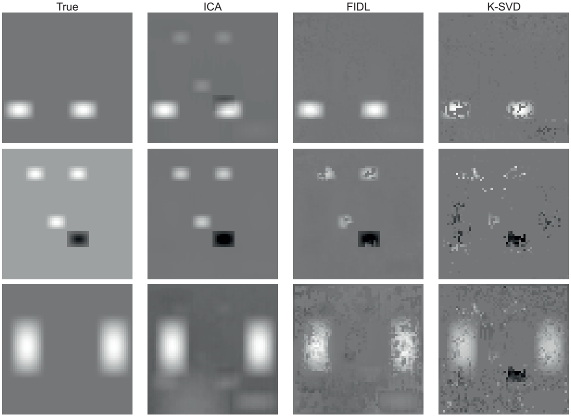

fMRI suited Dictionary Learning

Tackle several issues

- Spatial maps are possibly split
  - identify split maps
  - merge them back
- time courses can get interrupted.
- The dense characteristics of the artifacts need to be incorporated
- Care about computational complexity

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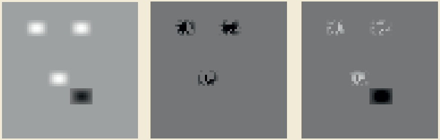
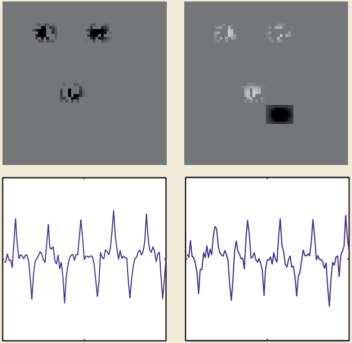
Example of split spatial maps



fMRI suited Dictionary Learning

Detect split atoms and merge them back

- Split maps corresponding to the same FBN are functionally associated, therefor should exhibit similar activation time-courses



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- **Spatial Maps Merging:**

1. Construct, $\boldsymbol{m}$, the vector with indices, say ρ of a set of highly correlated time courses.
2. Set, $\boldsymbol{\beta}$ the union of the supports of the spatial maps indexed in $\boldsymbol{m}$.
3. Solve for $\boldsymbol{a}$ and $\boldsymbol{b}$ the optimization problem

$$\min_{\boldsymbol{a},\boldsymbol{b}} \|(\boldsymbol{A}_{\cdot,m_1}\boldsymbol{B}_{m_1,\boldsymbol{\beta}} + \cdots + \boldsymbol{A}_{\cdot,m_\rho}\boldsymbol{B}_{m_\rho,\boldsymbol{\beta}}) - \boldsymbol{a}\boldsymbol{b}^T\|_{\mathsf{F}}^2.$$

   Rank-1 best approximation (via truncated SVD).

4. Set $\boldsymbol{A}_{\cdot,m_1} = \boldsymbol{a}$ and $\boldsymbol{B}_{m_1,\boldsymbol{\beta}} = \boldsymbol{b}$
5. Re-initialize the rest of the atoms indexed in $\boldsymbol{m}$.

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- **Time Courses Merging:** It can be done in exactly the same way.

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- **Time Courses Merging:** It can be done in exactly the same way.
- Merging tasks need not to be performed in each K-SVD iteration.

fMRI suited Dictionary Learning

Coping with machine-induced artifacts

- These artifacts exhibit low localization rendered, in this way, dense.
- Force a number of rows of $\boldsymbol{B}$ to be dense: $\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{B} + \bar{\boldsymbol{A}}\bar{\boldsymbol{B}}$
- Sparse coding with Partly known support.

Computational Complexity issues

- Replace OMP with Batch OMP [Rubinstein 2008]:
  - Batch OMP Pre-computes $\boldsymbol{A}^T\boldsymbol{A}$ avoiding the explicit computation of the error residual in each iteration.
  - A large number of columns of $\boldsymbol{Y}$ (voxels) need to be sparse coded over the same dictionary.
  - t is larger than the number of dictionary atoms.
- Replace exact rank-1 solutions of $\min_{\boldsymbol{a},\boldsymbol{b}} \|\boldsymbol{\Gamma} - \boldsymbol{a}\boldsymbol{b}^T\|_{\mathsf{F}}^2$, with approximates:
  - e.g., with alternating optimization over $\boldsymbol{a}$ and $\boldsymbol{b}$ according to:

$$\boldsymbol{a} = \frac{\boldsymbol{\Gamma}\boldsymbol{b}}{\|\boldsymbol{\Gamma}\boldsymbol{b}\|_2}, \qquad \boldsymbol{b} = \boldsymbol{\Gamma}^T\boldsymbol{a}$$

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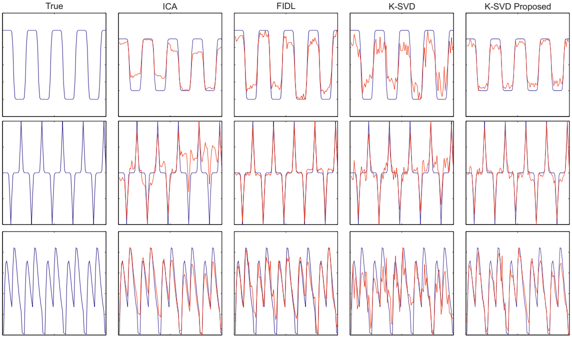

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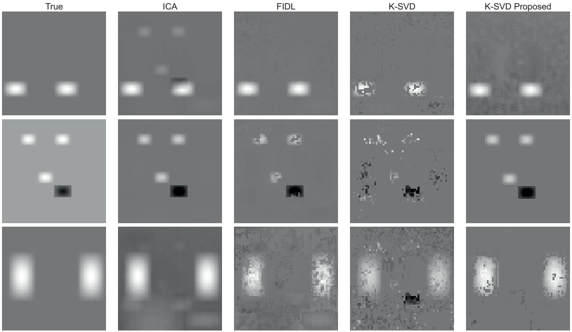

Thank you

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